# Appendix F: Guidance to the MATLAB code

The consumption-based account calculation can be broken down into three principal parts to be implemented mathematically separately:

1. The allocation of Norwegian production emissions from industrial sources to final products produced in Norway.
    a. This is essentially the allocation of the Norwegian production-based emissions through the Norwegian input-output table (representing Norwegian supply-chains) to Norwegian final demand.
2. The estimation of emissions embodied in imports used in intermediate production of the Norwegian economy.
    a. This is the emissions embodied in the import of goods/services used by Norwegian industry. For example, emissions associated with foreign feed production used by the Norwegian agricultural industry would be included here.
3. The estimation of emissions embodied in imports imported directly to final consumers.
    a. This is the emissions embodied in the imports of goods/services purchased directly by Norwegian consumers – for example, the emissions embodied in vehicle or consumer electronic device that is wholly produced overseas.

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Some prior knowledge is required for the understanding of the background to the model and of input-output in general. A more detailed write-up behind the concept of the model is available at: https://doi.org/10.5281/zenodo.1489942. A list of variables is included in the section "Notation" below.

As above, the footprint can be calculated as the impacts from the domestic model in domestic final demand, plus the impacts embodied in imports (separately to intermediate producers and final consumers).

## Norwegian domestic emissions

The Norwegian IO model is the mechanism for the allocation of Norwegian production emissions to final goods, as well as for the allocation to final demand of the emissions embodied in imports which are used in intermediate production of the Norwegian economy. Both are data downloadable from Norwegian statistics. Note that the data from Norwegian statistics only shows emissions at a breakdown for 46 sectors. In order to align the classification of the data to the Norwegian Input-Output table, these 46 sectors must be disaggregated to the 64 sectors of the IO table. The data reported in the Eurostat data portal does not provide any additional resolution. A test was done to use EXIOBASE data to disaggregate, but EXIOBASE is similarly based on assumptions for the higher resolution needed here, and the data showed clear incompatibilities. As a result, the raw emissions data was disaggregated proportionally to output, such that all disaggregated sectors would have the same emission intensities. The sectors impacted are mostly service sectors, so the assumption there is that these sectors have the same direct emission intensity. Since most of the carbon footprint of these sectors are in their supply-chains, and not direct emissions, this assumption should not have a major effect on resultant multipliers.

The only additional data used in this model is the inclusion of capital goods alongside normal goods in the intermediate inputs of goods into production. The data on the use of capital comes from the EXIOBASE dataset, and is aggregated to the Norwegian classification for the purposes here.

The impacts from the domestic model in domestic final demand are calculated as:

$$D^d = s^d\left(I - \widehat{(A^d)}\right)^{-1} Y^d \qquad \textit{Equation 1}$$

Including capital this becomes

$$D^d = s^d\left(I - \widehat{(A^d + K^d)}\right)^{-1} Y^d \qquad \textit{Equation 2}$$

Where $D^d$ is the domestic component of the Norwegian footprint, $s^d$ is the emissions intensity of Norwegian production (Norwegian emissions by sector divided by output by sector), $I$ is an identity matrix (a matrix of ones along the diagonal), $A^d$ is the technical coefficients of the domestically produced output used by Norwegian industries, $K^d$ is the technical coefficients of the domestically produced capital used by Norwegian industries, $Y^d$ is the final demand of Norwegian produced output.

The domestic coefficients are calculated as:

$$A^d{}_{.j} = \frac{Z_{.j}}{x_j} \qquad \textit{Equation 3}$$

Where $A^d{}_{.j}$ are the coefficients of each intermediate input, by sector j, $Z_{.j}$ are the actual Norwegian intermediate inputs (domestically sourced), by sector j and $x_j$ is the industrial output by sector j.

The domestic capital coefficients are calculated as:

$$K^d{}_{.j} = \frac{Z^{Kd}{}_{.j}}{CFC_j} \qquad \textit{Equation 4}$$

Where $K^d{}_{.j}$ are the coefficients of each capital input, by using sector j, $Z^{Kd}_{.j}$ are the values of capital produced in Norway used by Norwegian sectors (as reported by EXIOBASE), by sector j and $CFC_j$ is the consumption of fixed capital by sector j as a coefficient. The CFC in absolute terms is available from the Norwegian IO tables, and is divided by output $x_j$ to transform to coefficients.

Define the Leontief inverse matrix and shows the amount of Domestic production required to fulfill a unit of Norwegian final demand.

$$L^d = \left(I - \left(A^d\right)\right)^{-1} \qquad \textit{Equation 5}$$

And with capital endogenised:

$$L^{dk} = \left(I - \left(A^d + K^d\right)\right)^{-1} \qquad \textit{Equation 6}$$

and with a diagonal hat shows diagonalisation to keep the sector dimension of the calculation:

$$D^d = (\widehat{s^d L^d}) Y^d \qquad \textit{Equation 7}$$

Gives the domestic emission footprints for each product. Similarly, including capital:

$$D^{dk} = (\widehat{s^d L^{dk}}) Y^d \qquad \textit{Equation 8}$$

## Emissions embodied in imports – coupled approach

For the impacts embodied in imports, the calculation stems from the basic economic balance where total imports $\boldsymbol{m}$ is the sum of intermediate imports $\boldsymbol{A^m x}$ and the sum over the k categories (columns) of final imports $\boldsymbol{Y^m}$:

$$\boldsymbol{m = A^m x + \sum_k Y_{.,k}^m} \qquad \textit{Equation 9}$$

Note that $\boldsymbol{A^m x}$ is imports to Norwegian producer and $\boldsymbol{Y^m}$ is import of products purchased directly by the final consumer. To calculate the emissions embodied in imports, we need to know the emissions embodied in imports per unit of import. This information comes from the EXIOABSE model, and is referred to as the MRIO multiplier, denoted $\boldsymbol{Q^{imp,64}}$. There are a few steps involved in arriving at the multiplier relevant for Norwegian imports, and in the Norwegian industry classification – further notes are provided on this topic below in the section "Multipliers for emissions embodied in imports from EXIOBASE".

The multiplier $\boldsymbol{Q^{imp,64}}$ shows the emissions per unit of imported goods/services and can simply be multiplied by the economic value of the imports, which gives $\boldsymbol{D^m}$

$$\boldsymbol{D^m = Q^{imp,64} m} \qquad \textit{Equation 10}$$

Of note, the emissions embodied in imports corresponds to the goods/services imported into Norway, regardless of who imports the goods/services. For example, the emissions embodied in imports corresponding to the agricultural sector would include feed import to the agricultural sector, as well as agricultural production such as grains used directly by households, the food manufacturing sector, and other industries.

In order to allocate the emissions embodied in imports to the final goods and services consumed by Norwegian residents, we must apply the import balance of Equation 10 to break down information about which intermediate user $\boldsymbol{A^m x}$ or final user $\boldsymbol{Y^m}$ imports the goods/services.

That is,

$$\boldsymbol{D^m = Q^{imp,64} m = Q^{imp,64}(A^m x + Y^m)} \qquad \textit{Equation 11}$$

$\boldsymbol{Q^{imp,64} A^m}$ shows the total imported emissions per unit of Norwegian production for intermediate use of imports. It is a matrix multiplication between the variable $\boldsymbol{Q^{imp,64}}$ specified in Equation 22 and the intermediate import component of the coefficient matrix "A".
Now, as $\boldsymbol{x = \left(I - \left(A^d\right)\right)^{-1} Y^d}$ (from basic input-output relationships), we can substitute to get:

$$\boldsymbol{D^m = Q^{imp,64} m = Q^{imp,64}\left(A^m \left(I - \left(A^d\right)\right)^{-1} Y^d + Y^m\right)} \qquad \textit{Equation 12}$$

And simplifying using the notation for the Leontief inverse as above:

$$\boldsymbol{D^m = Q^{imp,64} A^m L^d Y^d + Q^{imp,64} Y^m} \qquad \textit{Equation 13}$$

$\boldsymbol{Q^{imp,64} A^m L^d}$ is a matrix multiplication between the matrix $\boldsymbol{Q^{imp,64} A^m}$ (which shows imported emissions per unit of Norwegian production) and $\boldsymbol{L^d}$ (which shows the total Norwegian production required per unit of final demand).

Equation 13 is thus allocating the emissions embodied in imports $\boldsymbol{D^m}$ (on the left hand side) to the final goods/services of Norwegian finally produced goods ($\boldsymbol{Y^d}$) and imports direct to final demand ($\boldsymbol{Y^m}$) – on the right hand side.

The total environmental footprint from industrial sources is the summation of Equation 7 and 13, and make the total Norwegian footprint for goods and services as:

$$\boldsymbol{D^{d+m} = s^d L^d Y^d + Q^{imp,64} A^m L^d Y^d + Q^{imp,64} Y^m} \qquad \textit{Equation 14}$$

A full derivation of the inclusion of capital is not provided, but follows the above derivation. The differences are that the domestic use and import of capital goods used in Norwegian production is included, and the import multipliers from EXIOBASE include capital. As such:

$$\boldsymbol{D^{(d+m)k} = s^d L^{dk} Y^d + Q^{impK,64} A^{mK} L^d Y^d + Q^{impK,64} Y^m} \qquad \textit{Equation 15}$$

Where the import capital coefficients are calculated as:

$$\boldsymbol{K^m{}_{.j} = \frac{Z^{Km}{}_{.j}}{CFC_j}} \qquad \textit{Equation 16}$$

Where $\boldsymbol{K^m{}_{.j}}$ are the coefficients of each capital input, by using sector j, $\boldsymbol{Z^{Km}_{.j}}$ are the values of capital produced overseas used by Norwegian sectors (as reported by EXIOBASE), by sector j and $\boldsymbol{CFC_j}$ is the consumption of fixed capital by sector j as a coefficient. The CFC in absolute terms is available from the Norwegian IO tables, and is divided by output $\boldsymbol{x_j}$ to transform to coefficients.

$\boldsymbol{A^{mK}}$ t is then defined as:

$$\boldsymbol{A^{mK} = A^m + K^m} \qquad \textit{Equation 17}$$

## Emissions embodied in imports - Multipliers derived from EXIOBASE

For Equation 10 and onwards, emission "multipliers" are required, which show the upstream life-cycle emissions per unit of imports. These multipliers are extracted from the EXIOBASE dataset, but require reclassification (due to differing industry classifications between EXIOBASE and the Norwegian IO table) before implementation in the Coupled model (Equation 10 and onwards).

The re-classified multipliers (in Norwegian IO classification) are generated by the division of absolute values of emissions embodied in imports by the economic value of imports in order to obtain emission multipliers per unit of imports. This division is done on derived accounts in absolute terms, as the reclassification involves aggregation across multiple industries. Aggregation can be performed directly for values in absolute terms, but not for intensity values such as multipliers.

Two sets of data are needed from EXIOBASE:

1. Emissions embodied in imports – these are the full supply chain emissions to point of purchase (i.e. total upstream life-cycle emissions) of sector imports as calculated via the EXIOBASE MRIO. Hence, the value in "Cattle farming" will include all supply chain emissions of cattle farming imports, including the emissions released in cattle farming imports directly, and any emissions associated with feed production used in cattle farming.

2. Value of imports. Reported in million Euro in the EXIOBASE database - these are the total imports into Norway as reported in EXIOBASE. Please note, that the import into Norway from EXIOBASE does not necessary equal the import into Norway according to Statistics Norway. This is due to the fact that global import = global export in a global MRIO as EXIOBASE, and to make sure this is the case the trade data in EXIOBASE is re-balanced[1]. In addition, re-exports (imports that are directly exported without any transformation by Norwegian producers are excluded from a MRIO calculation). As the import-value from EXIOBASE is used only as a weight in the calculation of the emission multipliers from EXIOBASE, the re-balancing is not seen as an issue that affects the carbon footprint in the coupled model. For full details on the handling of trade data in EXIOBASE, the reader is referred to Stadler et al 2018.

The multipliers derived from EXIOBASE are produced by the function ExtractMultipliers.m. All steps are documented in that Matlab script. Multipliers including and excluding capital are calculated.

These two sets of data are then aggregated to the 64 sector classification for Norway, using the concordance matrix "EXIOBASEtoNO64". Note the concordance matrix should be as precise as possible – the purpose of it is to allocate the most relevant multiplier from EXIOBASE to the most relevant Norwegian industry. In practice, it is likely that there are a number of many-to-many, many-to-one and one-to-many links in the concordance matrix. This does not present a problem mathematically, but has some conceptual implications:

- In the case of one EXIOBASE sector linking to many Norwegian sectors, the Norwegian sectors will each be assigned the same EXIOBASE multiplier – for example "Hotels and restaurants" is one sector in EXIOBASE, whilst it is split in the Norwegian IO table. As EXIOBASE does not provide any more detail on differences in emissions intensity between Hotels and Restaurants, the best we can do is assign the aggregate emissions multiplier to both sectors.
- In the case of many EXIOBASE sectors linking to one Norwegian sector. This occurs for agriculture, where EXIOBASE has 15 agricultural industries, and Norwegian one. As the Norwegian IO model can only treat the aggregate import of agricultural goods, the EXIOBASE data must be aggregated. Hence a simple aggregation of EXIOBASE emissions embodied in imports and import value is taken to give a single multiplier for the Norwegian model. Note that the multiplier is a "weighted" multiplier of the 15 EXIOBASE sectors based on import value.
- In the case of many EXIOBASE sectors linking to many Norwegian sectors, both the above steps are implemented together – all EXIOBASE emissions embodied in imports for the relevant sectors are aggregated, and the same is done for import value. The emission multiplier is calculated as the division of the aggregated emissions embodied in imports by the aggregated imports; then the emission multiplier is assigned to all relevant Norwegian industry sectors.
- When there are one-to-many or many-to-many linkages, the intermediate steps of calculating aggregate "emissions embodied in imports" and "imports" will show double counting of values such that if you sum the value of the aggregated imports over the whole economy, it will be larger than known imports. However, this double counting occurs consistently for both imports and footprints, such that the calculation of multipliers cancels the double counting out.

Mathematically we can express this as:

$\mathbf{Q}^{\mathbf{imp}}$ – the emission multipliers in EXIOBASE classification; dimension 163 industries. Note these multipliers are calculated explicitly for Norwegian imports.
$\mathbf{D}^{\mathbf{imp}}$ – the value of emissions embodied in imports, as extracted from EXIOBASE; dimension 163 industries.

[1] https://zenodo.org/record/4588235

m – the value of imports to Norway, as extracted from EXIOBASE; dimension 163 industries. $\hat{\mathbf{m}}$ signifies diagonalisation

Now, using a ′ to signify matrix transposition,

$$\boldsymbol{D^{imp'} = Q^{imp}\hat{m}} \qquad \textit{Equation 18}$$

Or alternatively,

$$\boldsymbol{Q^{imp'} = D^{imp'}\hat{m}^{-1}} \qquad \textit{Equation 19}$$

As this data is in EXIOBASE classification, we introduce a concordance matrix as discussed above:
G – the relationship between EXIOBASE 163 and Norwegian 62 industries. Dimension 163 EXIOBASE industries by 64 Norwegian industries.

Now we aggregate emissions embodied in imports and imports separately:

$$\boldsymbol{D^{imp,64} = [\![(D]\!]^{imp'} * G)'} \qquad \textit{Equation 20}$$

$$\boldsymbol{m^{64} = [\![(m]\!]' * G)'} \qquad \textit{Equation 21}$$

And calculate the aggregated multipliers, the same as in equation 19:

$$\boldsymbol{Q^{imp,64} = \left(D^{imp,64'} * (\widehat{m^{64}})^{-1}\right)'} \qquad \textit{Equation 22}$$

Gives us the multipliers in Norwegian classification. The «import» multipliers cover the supply chain to the point of import into Norway. The same derivation is done for import multipliers including capital, defined as:

$$\boldsymbol{Q^{impK,64} = \left(D^{impK,64'} * (\widehat{m^{64}})^{-1}\right)'} \qquad \textit{Equation 23}$$

Where $\boldsymbol{D^{impK,64'}}$ is the aggregated carbon footprints as calculated by EXIOBASE with capital endogenised.

## Notation

Variables used in the derivation are defined as:

| Variable | Name | Description |
|---|---|---|
| I | The identity matrix | The Matrix "I" has "1" in the diagonal and the number 0 outside the diagonal and it is only used in the calculation of the Leontief Inverse |
| $\boldsymbol{A^d}$ | Coefficient matrix - domestic | Inter-industry coefficients of domestic transactions (direct requirements matrix), dimension n x n |
| $\boldsymbol{A^m}$ | Coefficient matrix - imports | Inter-industry coefficients of import transactions (direct requirements matrix), dimension n x n |
| $\boldsymbol{K^d}$ | Capital coefficient matrix - domestic | Inter-industry coefficients of domestic capital (direct requirements matrix), dimension n x n |

| | | |
|---|---|---|
| $\boldsymbol{K^m}$ | Capital coefficient matrix - imports | Inter-industry coefficients of import capital use (direct requirements matrix), dimension n x n |
| $\boldsymbol{A^{mK}}$ | Coefficient matrix | Inter-industry coefficients of import transactions (both current and capital) (direct requirements matrix), dimension n x n |
| $\mathbf{L}^d$ | Leontief inverse | Leontief inverse (total requirements matrix), Domestic transactions only, dimension n x n |
| $\mathbf{Y}^d$ | Final demand, domestic | Final demand matrix of domestically produced goods (sectors and final demand categories), dimension n x r |
| $\mathbf{Y}^m$ | Final demand, imports | Final demand matrix of imported goods (sectors and final demand categories), dimension n x r |
| $\mathbf{x}$ | Gross output | Total output of industries, dimension n |
| $\mathbf{m}$ | Imports | Monetary imports, dimension n x n |
| $\boldsymbol{s^d}$ | Emission coefficients | Emissions per unit output of production sectors, domestic sectors only, data made available as a matrix of air emissions, but only 1 category of air emission modelled at a time, hence dimension 1 x n. |
| $\mathbf{D}$ | Footprints | Footprint of final demand |
| $\boldsymbol{Q^{imp,64}}$ | Multipliers | Emissions per unit of final demand, superscript signifies that they relate to Norwegian imports, in 64 sector classification |
| $\boldsymbol{Q^{impK,64}}$ | Multipliers | Emissions per unit of final demand, superscript signifies that they relate to Norwegian imports and include endogenized capital, in 64 sector classification |
| i,j | | Industries 64 sectors in the Norwegian model |
| n | | Air emission categories |
| K | | Used as a superscript for capital |